MATH 53, Practice for Midterm 2

You should allocate 90 minutes to do the following 9 problems (starting on the back of this page). The difficulty and spread of topics are *not* indicative of the actual midterm.

Make sure to show your reasoning, as an answer with no explanation will receive no credit on the actual exam. It is also a good habit to box your final answers.

Name: Student ID:

Problem 1. Explain why $f(x, y) = 2x^2 + y^2$ must have absolute extrema constrained to the circle $x^2 + y^2 = 1$, and find them with Lagrange multipliers.

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Problem 2. Evaluate the integral

$$\int_0^{\sqrt{2}} \int_{y^2}^{2} \frac{y}{1+x^2}\, \mathrm{d}x\, \mathrm{d}y.$$

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Problem 3. Find the volume of the region above the cone $z = \sqrt{x^2 + y^2} - 2$ and the below the paraboloid $z = 4 - x^2 - y^2$.

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Problem 4. Convert the following triple integral to Cartesian coordinates, but do not evaluate it.

$$\int_0^{\pi/2} \int_0^1 \int_{r^2}^r r^2 \cos\theta \, \mathrm{d}z \, \mathrm{d}r \, \mathrm{d}\theta.$$

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Problem 5 (Stewart Ch. 15 Review #55). Evaluate

$$\iint_R \frac{x-y}{x+y}\,\mathrm{d}x\,\mathrm{d}y$$

where R is the square with vertices $(0,2), (1,1), (2,2), (1,3)$.

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Problem 6 (Stewart §16.2.2)**.** Let C be the part of the curve $x^4 = y^3$ between the points $(1, 1)$ and $(8, 16)$. Compute $\int_C (x/y)\,\mathrm{d}s$.

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Problem 7.

a) Find a function $f(x, y, z)$ such that

$$\nabla f = \langle 3x^2yz - 3y, x^3z - 3x, x^3y + 2z \rangle.$$

b) Let C be the line segment starting at $(0, 0, 2)$ and ending at $(0, 3, 0)$. Evaluate

$$\int_C \langle 3x^2yz - 3y + \arctan(yz), x^3z - 3x + 2\cos(x^2), x^3y + 2z \rangle \cdot d\mathbf{r}.$$

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Problem 8. Let C be the ellipse $x^2/4 + y^2 = 1$, oriented counterclockwise. Briefly explain why Green's theorem **cannot** be directly applied to the integral

$$\int_C \left(y\log_4(x^2+4y^2) + 3x^2y^2\cos(x^3)\right)\mathrm{d}x + \left(-7x + 2y\sin(x^3)\right)\mathrm{d}y.$$

Find a way around this issue and evaluate the integral.

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Problem 9. Suppose we have an object of mass M occupying some region D, and let $CM = (\bar{x}, \bar{y})$ be its center of mass. If $P = (a, b)$ is any point in the plane, we can define the moment of inertia about P as

$$I_P = \int_D (\text{distance from } P \text{ to } (x, y))^2 \, \mathrm{d}m.$$

a) Calculate $I_P - I_{CM}$, leaving your final answer in terms of the constants $a, b, \bar{x}, \bar{y}, M$ only (there should be no integrals in your final answer).

b) Suppose that that our object is a circular cable of radius 2 centered at the origin with constant density and total mass $M = \pi$. Let $P = (1, 0)$. Find I_P. (Even if you did not solve (a), you can do this directly.)